Optimum Noncoherent Receiver at Low Signal-to-Noise Ratio for Unknown Doppler Shifted Signals

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An optimum low signal-to-noise ratio receiver is derived for the case when the received signal has unknown phase and an unknown doppler shift. This receiver appears to be new and is quite similar in form to the wideband frequency-shift-keyed receiver.

I. Introduction

The purpose of this article is to derive the optimum low-signal-to-noise ratio (SNR) noncoherent receiver when an unknown doppler shift is present on the received signal. This work was part of a low-data-rate receiver system study, with a possible application to Pioneer Venus.

The receiver derived here appears to be new and is quite similar to the existing noncoherent receiver (wideband frequency-shift-keyed (FSK) receiver) that has been used previously in radar systems as well as space telemetry applications where doppler is present. The wideband FSK receiver has two bandpass filters centered at the two possible transmitted tones and is wide enough in bandwidth to cover all possible doppler shifts. Following the filters, a square-law detector (or a linear envelope detector) is used, which is, in turn, fed into an integrate-and-dump circuit. The system is shown in Fig. 1 in a realization useful for comparing with the receiver developed in this article.

II. Signal Model

We now consider the signal model used to develop the optimum receiver. Motivated by the interest in low-data-rate systems, we consider an assumed received signal of the form

$$y(t) = A \sin [\omega_0 t + \Omega_j t + \omega t + \phi] + n(t)$$
$$0 \leq t < T, \qquad j = 1, 2 \tag{1}$$

where A is the received signal amplitude, ω_0 is the center frequency, Ω_j is the known frequency offset corresponding to signal j, ω is the unknown doppler offset frequency, ϕ is the unknown phase of the received signal, and $n(t)$ is white Gaussian noise with two-sided spectral density $N_0/2$. We assume that the time of occurrence of the received signal is known exactly.

In order to proceed with the receiver derivation, some assumption about the unknown doppler frequency and phase must be made. We assume here that ω is a random variable having a density function given by

$$p(\omega) = \frac{1}{2\omega_u} \qquad -\omega_u \leqq \omega < \omega_u \tag{2}$$

where ω_u is the maximum assumed value of the doppler shift. In addition we assume that ϕ is a uniform random variable on the interval $[-\pi, \pi]$. The optimum detection system is determined from the *a posteriori* probability that signal j was sent, given that $y(t)$ has been received, i.e., $P[j \mid y(t)]$. This can be obtained by computing $P(j \mid y(t), \phi, \omega)$ and averaging over ϕ and ω.

It has been shown by Viterbi (Ref. 1) and elsewhere that

$$P[j \mid y(t), \phi, \omega] = c \exp\left[\frac{2A}{N_0}\int_0^T y(t) \sin[\omega_0 t + \phi + \omega t + \Omega_j t]\, dt\right] \tag{3}$$

where c is a constant independent of j. Now with the assumption that the phase ϕ is uniform in $[-\pi, \pi]$ we have the result that

$$P[j \mid y(t)] = c\int_{-\omega_u}^{\omega_u}\int_{-\pi}^{\pi}\frac{1}{2\pi}\exp\left[\frac{2A}{N_0}\int_0^T y(t)\sin[\omega_0 t + \omega t + \Omega_j t + \phi]\, dt\right] d\phi\, p(\omega)\, d\omega \tag{4}$$

After making some algebraic manipulations and integrating on ϕ, we have

$$P[j \mid y(t)] = c\int_{-\omega_u}^{\omega_u} I_0\left[\frac{2A}{N_0}\sqrt{\iint_0^T y(t)\, y(s)\cos[\omega_0(t-s) + \Omega_j(t-s) + \omega(t-s)]\, dt\, ds}\right] p(\omega)\, d\omega \tag{5}$$

This result has been previously obtained using complex notation, by Ferguson (Ref. 2); however, he did not pursue this result to obtain the structure of the receiver other than to assert that at low signal-to-noise ratios it was quadratic. Equation (5) appears impossible to evaluate in closed form; however, with the assumption that we are concerned only with low signal-to-noise ratios, Eq. (5) with the use of Eq. (2) can be written as

$$P[j \mid y(t)] \cong c\left\{1 + \left(\frac{A}{N_0}\right)^2 \frac{1}{2\omega_u}\iint_0^T y(t)\, y(s)\int_{-\omega_u}^{\omega_u}\cos[\omega_0(t-s) + \Omega_j(t-s) + \omega(t-s)]\, d\omega\, dt\, ds\right\} \tag{6}$$

Performing the integration on ω yields

$$P[j \mid y(t)] = c + \left(\frac{A}{N_0}\right)^2 c\iint_0^T y(t)\, y(s)\cos[\omega_0(t-s) + \Omega_j(t-s)]\frac{\sin\omega_u(t-s)}{\omega_u(t-s)}\, dt\, ds \tag{7}$$

This result has been obtained by Raemer (Ref. 3), but the system to obtain the result was not given. A system that will perform the operation prescribed by Eq. (7) is given in Fig. 2.

In Fig. 2, $h_1(\tau)$ and $h_2(\tau)$ are the impulse responses and are given by

$$\left.\begin{aligned} h_1(\tau) &= \cos[\omega_0\tau + \Omega_1\tau]\frac{\sin\omega_u\tau}{\omega_u\tau}, \qquad \text{for all } \tau \\ h_2(\tau) &= \cos[\omega_0\tau + \Omega_2\tau]\frac{\sin\omega_u\tau}{\omega_u\tau}, \qquad \text{for all } \tau \end{aligned}\right\} \tag{8}$$

Since $h_1(\tau) \neq 0$ for all negative τ, we see that the receiver is not realizable. The transfer functions $H_1(f)$ and $H_2(f)$ associated with $h_1(\tau)$ and $h_2(\tau)$ are ideal bandpass filters. The structure of the ideal system is similar to the wideband FSK receiver except that only one filter per tone is used instead of two. The filtered signal branch can be viewed as an estimator of the incoming signal frequency so as to provide a phase- and frequency-coherent demodulation reference.

This optimum receiver requires ideal bandpass filters which introduce no delay in the signal. "Real" filters would introduce some delay but this delay could be equalized with a resulting increase in circuit complexity. Without delay compensation, the performance would be degraded.

We now show that if $\omega_u = 0$ the resulting receiver is the usual noncoherent receiver. Letting $\omega_u = 0$ in Eq. (7) and neglecting the constants, one has

$$D_j = \iint_0^T y(t)\, y(s) \cos\left[\omega_0 (t - s) + \Omega_j (t - s)\right] dt\, ds \tag{9}$$

Expanding the cosine, we have

$$D_j = \left(\int_0^T y(t) \cos(\omega_0 t + \Omega_j t)\, dt\right)^2 + \left(\int_0^T y(t) \sin(\omega_0 t + \Omega_j t)\, dt\right)^2 \tag{10}$$

which is the optimum decision statistic for all signal-to-noise ratios for the noncoherent receiver with zero doppler offset.

The ordinary noncoherent receiver can be constructed using a system similar to Fig. 2. From Eq. (9) we see that the system of Fig. 1 can be used if $h_1(\tau)$ and $h_2(\tau)$ are defined as (noting that the system will be unrealizable)

$$\left.\begin{aligned} h_1(\tau) &= \cos(\omega_0 + \Omega_1)\,\tau, \qquad \text{for all } \tau \\ h_2(\tau) &= \cos(\omega_0 + \Omega_2)\,\tau, \qquad \text{for all } \tau \end{aligned}\right\} \tag{11}$$

The spectrum associated with each impulse response of Eq. (11) is a pair of delta functions occurring at $\pm(\omega_0 + \Omega_1)$ and $\pm(\omega_0 + \Omega_2)$ respectively. In other words, only the component of energy exactly at $\omega_0 + \Omega_1$ and $\omega_0 + \Omega_2$ is passed to provide the reference.

Equation (6) can be rewritten to show that the optimum low signal-to-noise ratio system requires the computation of S_i; $i = 1, 2$, where S_i is given by

$$S_i = \int_{-\omega_u}^{\omega_u} \left\{\left(\int_0^T y(t) \cos\left[(\omega_0 + \Omega_i + \omega)\, t\right] dt\right)^2 + \left(\int_0^T y(t) \sin\left[(\omega_0 + \Omega_i + \omega)\, t\right] dt\right)^2\right\} \frac{1}{2\omega_u}\, d\omega$$

that is, the optimum low-SNR receiver with unknown doppler shift that can be viewed as a system to form spectral coefficients weighted with a uniform weighting.

References

1. Viterbi, A. J., *Principles of Coherent Communication*, McGraw-Hill, 1966.
2. Ferguson, M. J., "Communication at Low Data Rates—Oscillator Models and Corresponding Optimal Receivers," *IEEE Trans. Commun. Technol.*, Aug. 1968.
3. Raemer, H. R., *Statistical Communication Theory and Applications*, Section 7.1.4, Prentice-Hall, 1969.

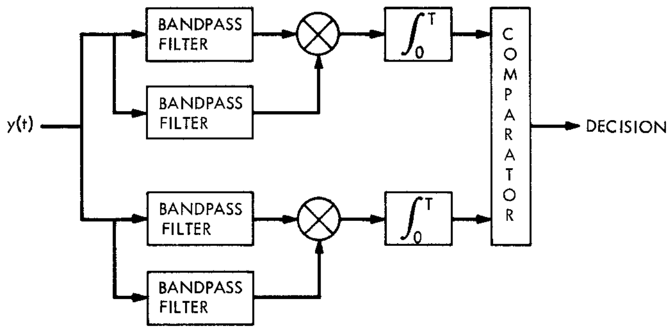

Fig. 1. Existing wideband FSK receiver

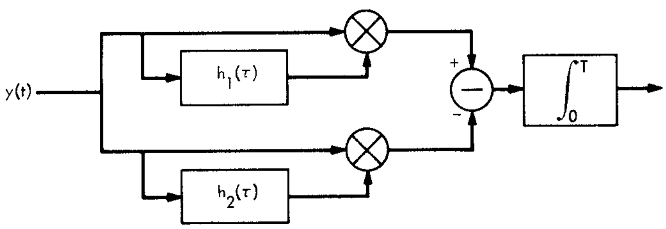

Fig. 2. Optimum low-SNR doppler-ambiguous receiver